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Living Mathematics

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Abstract

I define my 'living mathematics' as my living-educational-theory of teaching and researching mathematics. I define 'Living Mathematics' as the overarching values-based approach to the teaching and research of mathematics as a parallel to the distinction made between 'living-educational-theory' and 'Living Educational Theory research'. In this article I ask the question 'how do I improve my practice of teaching and researching here?' by exploring how I:

(1) As a teacher can support mathematical thinking and the understanding of textbook concepts using a value-based approach and,
(2) As a researcher can enhance my mathematical thinking and modify, or create, mathematical models by calling upon my lived experiences, capturing and representing them in a symbolic form.

I define teaching and research pathways in Living Mathematics as sequences of useful and focused key actions. Four exemplar case studies of my living mathematics are discussed; two from the teaching pathway and two from the research pathway.

Keywords: Mathematics Education; Living Educational Theory; Mathematical thinking; Integrative education.

Introduction

In this paper I explore my narrative around my teaching and scholarly practice in mathematics. It uses myself as a living contradiction, reporting on a dissonance between my values and beliefs and my actions. As I create this paper my intention is to help me 'improve my practice here' (Whitehead, 1989, 2019).

Living Mathematics is a values-based approach to mathematics teaching and research. It is also a teaching strategy and a research methodology.

The approach is interdisciplinary and sets out to connect ideas and concepts across the boundary between Living Educational Theory and mathematics. At this boundary the values-based approach may interact with mathematical forms. For example, I may simply believe that addition (+) is good and subtraction (-) is bad. Further more the square root of the area of a square (for example, 25) is the length of its side (5): a very down-to-earth, concrete and pragmatic result. However, I could encounter a living contradiction with respect to my value 'pragmatism' when asked to consider the square root of -1. This is because the square root of –1 is abstract and cannot be represented physically as the length of a line. On the other side of this argument I could encounter a living contradiction with respect to my value 'completeness' if no representation of the square root of -1 were available.

Another example of how Living Educational Theory and mathematics interact can be found in geometry. I could be overcome by a sense of awe and wonder when visualising the base angle in a right-angled triangle slowly increasing from 1 degree to 10, 20, 40, 80, 89, 89.9, 89.99 degrees... towards 90 degrees. I know that when it reaches 90 degrees the hypotenuse and the opposite side of the triangle will be parallel. Some may argue that the apex of the triangle has disappeared. Some may argue that the shape is no longer a triangle anyway. I could keep on reaching out to try and touch the apex that has disappeared, in line with my value 'perseverance', and believe that this would be possible, in line with my value 'faith': or perhaps I would not.

I will develop my thesis by firstly conceptualising my 'I' as four mutually exclusive worlds. The overarching Living Mathematics and the relatable 'living mathematics' will be defined along with two pathways of key actions, one for teaching and one for research. I consider this central to use of the Living Educational Theory research approach here.

Four case studies will then be described:

a. I as a drama in mathematics teacher,

b. I as a teacher of morals in mathematics,

c. I as the author of a five-cycle living visual taxonomy of learning interactions,

d. I as a generalisor of formulae for estimating heritability.

All four case studies focus on my narrative and show how my living mathematics can be in my teaching and in my research but differently.

I will then discuss to what extent writing this paper has helped me to gain more understanding of:

i) mathematical thinking,
ii) teaching and research pathways,
iii) the living context of a mathematical form,
iv) my value integrity and damned lies,
v) reverence and authority,
vi) what it means to be 'only human',
vii) play and drawing pictures,
viii) taking a toy to pieces, and
ix) 'I' as my claim to knowledge

My 'I'

Throughout my early life I focused on play and drawing pictures. Later I experienced play, art and mathematics. I believe that it is essential to play with ideas, consistent with my values growth, perseverance and integrity, (Williamson, 2015); and that doodling most days throughout the last five decades has worked towards defining my art, and my personal visual subculture (Chalmers, 2019). I give myself permission to play with any set of symbols, diagrams, free form yet obedient shapes, colours, voices or sounds and to give a title, that I have made meaning of, to many of these creations. Further I hope that reflecting on my work through a mathematical lens has enriched my lived experience and practice as a teacher and researcher.

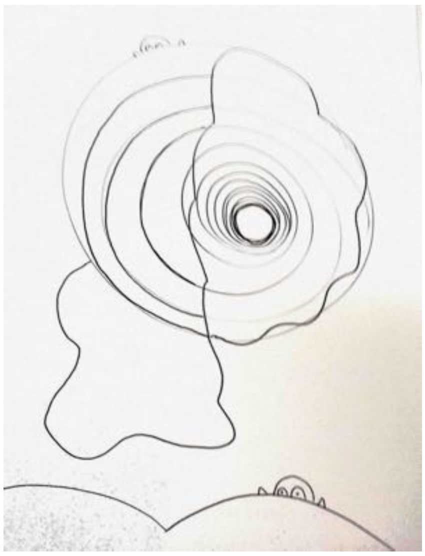

Figure 1a. 'People' 2006. **Figure 1b.** 'Monster Flower' 2012.

To me something that is mathematical is anything that I perceive to be inherently logically connected (Boaler, Chen, Williams and Cordero, 2016). For example, my doodle, 'People' 2006 (Figure 1a), shows a two-piece face jigsaw with no intersection and whose union equals the whole space. 'Monster Flower' 2012 (Figure 1b), shows an incoherent relativity, that is, a flower dwarfing two human forms, a "weirding" (Appelbaum, 2017) of my belief, or 'living pseudo-mathematical axiom', that people are bigger than flowers. Appelbaum suggests that this sort of environmental manipulation, or transient distortion of one's beliefs is both an attractor and motivator and a 'part of a science of pedagogy that constructs the need for a technology of attention' (Appelbaum, 2017, Chapter 3). It is intended to make us think and ask questions. Is it a rare type of giant flower? Is the flower big or very close or small or far away? It seems that the relationship between my mental world of meaning (Tall, 2004) and mathematics is negotiable. 'Most mathematicians have withdrawn from the world to concentrate on problems generated within mathematics' (Kline, 1982, page 278, Lakoff and Nunez, 2000) but I do not believe that this is what I have done.

I now understand that my 'I' as my claim to knowledge may exist in four worlds, each the creator of a life affirming energy, but each with its own special characteristics:

World One

… the mathematics that I consider to represent my living values

'my I' AND 'mathematics'

Not all of mathematics will do this. On first inspection of any mathematical form it may seem that none of my values and beliefs surround it and my senses towards it are numb, null. It is inanimate, however, my related values and beliefs may emerge as I reflect on its form, its meaning and my attitude and behaviour towards it. For example, my reflections may allow me to see the inanimate form:

$$1 + 1 = 2 \qquad (1)$$

as a beautiful form (Breitenbach, 2013) that uses a symbolism steeped in history (Chrisomalis, 2010), meaning yin and yang, or representing two birds on a perch in my mind. A form that helps me to feel unity or a type of perfection expressible in pros, verse or in a plethora of multisensory ways. At a more advanced level, my reflections may allow me to see the inanimate form:

$$y=1/x \qquad (2)$$

as a mysterious experience, a journey into the unknown. As x increases y becomes infinitesimally small but never reaches zero and 'never' is a long time. This could be visualised as an airplane coming into land but its wheels never touching the runway, albeit surreal, but perhaps illustrating a spiritually in me (Winter, 2001). Alternatively, as x decreases and approaches zero, y becomes larger and larger but as soon as $x = 0$, y becomes positive infinity and negative infinity at the same time. Perhaps this quantum Schrödinger's-cat type thought experiment raises more questions than it answers and gives me an opportunity to identify my values and beliefs that surround it.

World Two

…. the mathematics that does not represent my living values

'not my I' AND 'mathematics'

World two is relevant to my living mathematics because it represents mathematical forms that may be unheeding to the Living Mathematics approach. It seems that some mathematics is difficult or impossible to reach as I do in my first world. It seems to be part of another world that is disconnect from my 'I'. For example, my reflections may not allow me to see the inanimate form:

$$xy/w - c \qquad (3)$$

as anything other than inanimate unless it was known to represent an emotive application. I believe that this could occur when I feel that the mathematics is remote to me and part of someone else's world; perhaps as it would appear to a disengaged student. This could happen because I have not given time to reflect on its form, invent toy applications or to wrap it in the values and beliefs I may hold about the outcomes it may determine for me.

If any sound, shape or form that conveys meaning, feelings, values or beliefs is a language then all mathematics is a language, as is Spanish or English. The messages implicit may be personal and require an artistic voice to tease them out but nevertheless they are messages with meaning. If this conjecture is true then my second world is empty. I would conclude that the Living Mathematics approach has the potential to be applied to any mathematical form.

World Three

… part of my I that cannot be represented by mathematics

'my I' AND 'not mathematics'

I cannot believe that my whole being, including my awareness of others and all my values and believes stem from a mathematical form.

My third world is filled with many memories and ideas that are in no way mathematical. However, from time to time, I can ask myself the question, 'can what I am doing or observing here be represented using mathematics?' Often my answer is no.

World Four

… neither present-day mathematics nor my I

'not my I' AND 'not mathematics'

I believe that there is still more but it is hidden.

A conceptual framework

My simple four-worlds conceptual framework for my claim to knowledge can be represented by a Venn Diagram with two intersecting sets: 'my I' and 'mathematics'. I need to mention each of my four worlds in order to accommodate my value completeness.

Definitions

My living mathematics

Now I will deliberate on my first world, the mathematics that I consider to represent my living values (my I), which I refer to as my 'living mathematics', my living educational theory of teaching and researching mathematics.

Living Mathematics

I will refer to the intra-personal overarching values-based approach to teaching and research in mathematics as 'Living Mathematics', capitalised in order to mimic the distinction made between 'living educational theory' and 'Living Educational Theory research'.

I define a Living Mathematics 'teaching pathway' and a 'research pathway' as sequences of Key Actions that could be undertaken by others or myself. The actions involve making meaning (Kegan, 1980) and the identification of an individual's living values and beliefs (Whitehead, 1989).

Values and beliefs can either be (a) identified independently of a mathematical form or (b) identified in response to an individual making meaning of a mathematical form. My lived relatable experience is that option (a) corresponds to research (in a broad sense) because here the practitioner's values and beliefs are over arching and underpin the work. Alternatively option (b) corresponds to teaching because here the identification of values and beliefs may occur in response to a mathematical form: akin to a reaction expressed by a visitor to an art gallery.

Teaching Pathway

Key Action 1

Student and teacher becoming aware of published content

Key Action 2

Make meaning

Key Action 3

Identifying values and beliefs in response to a mathematical form

Key Action 4

Use of a life affirming energy to understand and teach

Research Pathway

Key Action 1

Identifying values and beliefs independently

Key Action 2

Researcher becoming aware of published content or an event

Key Action 3

Making meaning through a Living-Theoretic lens

Key Action 4

Use of a life affirming energy to modify or create new mathematics.

The case studies given next will illustrate the how these pathways can be actioned and the subtle differences between them.

Case Studies a and b: teaching pathways, c and d: research pathways

a) I as a drama in mathematics teacher

Following my teaching pathway.

Becoming aware of the content:

+3+4=7

-3 – 4 = - 7

-3+4=1 and

+ 3 – 4 = - 1

is to perform Key Action 1. Content that could perhaps be found in any mathematics textbook in a chapter on addition of positive and negative numbers.

Traditionally, a number line is used to make meaning of equations such as: '-3-4=-7 and +3-4=-1'. The positive sign meaning forwards (or making steps to the right) and the negative sign meaning backwards (or making steps to the left). The equations then translate into movement, like a dance on a line, and meaning is made following Key Action 2 to support the learning via this mathematical form.

Contextualising one's living values in this content (Key Action 3) may seem challenging at first but I will suggest here that this action is supported by not only the Living Educational Theory literature itself (for example, Bruce Ferguson, 2015) but also through the Socratic method. The values and beliefs held by a learner may be explored through a Socratic dialogue. Further, explorations and deliberations into a fantasy world (Lee, Lee and Lau, 2003) may support learning further.

Using the Socratic Method

Students may seek and gain a deeper understanding of concepts in a text through thoughtful dialogue rather than memorizing information that has been provided for them. The Socratic approach would use a series of questions and answers, ad libitum or scripted, to examine the text in detail, and perhaps such a dialogue would necessarily be underpinned by the participants living values and beliefs.

Using fantasy-based learning

Thoughtful dialogue as a creative pursuit may involve learner engagement with toy problems, that is, problems embedded in a near-real or fantasy situation, (Lee., Lee and Lau, 2003). I argue that such an exercise would involve locating the fantasy of the student and building upon it: a teaching strategy that could be used at all levels provided there is a visual or narrative 'hook' to hang the fantasy onto, necessitating the composition of pretend values and pretend beliefs and their contextualisation in the content. The ownership and authorship of the fantastical hook may be important determinants of learning outcomes (Richet and Schlesinger, 2016).

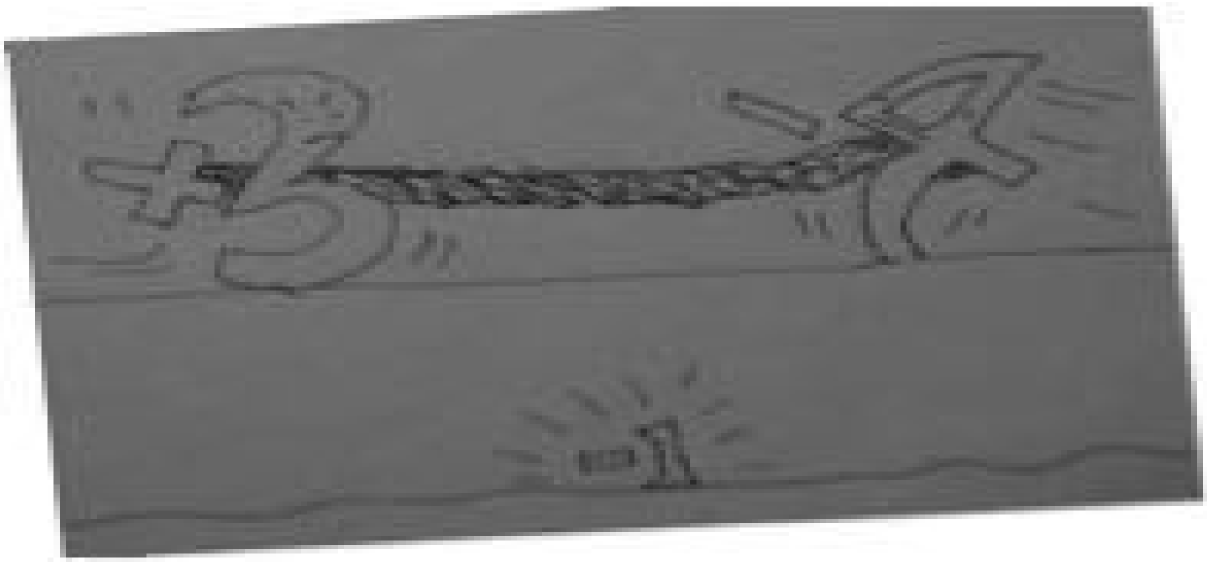

Figure 2. 'Tug of War +3 – 4 = - 1' 2016.

Devising a script.

Numbers are not people. Numbers do not play the game 'tug 'o war' (Figure 2), however, the personification of number may support Socratic enquiry and fantasy-based learning and provide a platform on which to build a deeper understanding. Personifying numbers manipulates the learning environment, as in Figure 1b, and this presents an opportunity to view the scene through a range of value-based lenses. Is '+' good and '-' bad? Does '4' being greater than '3', suggest that it is also stronger, more dynamic or more determined?

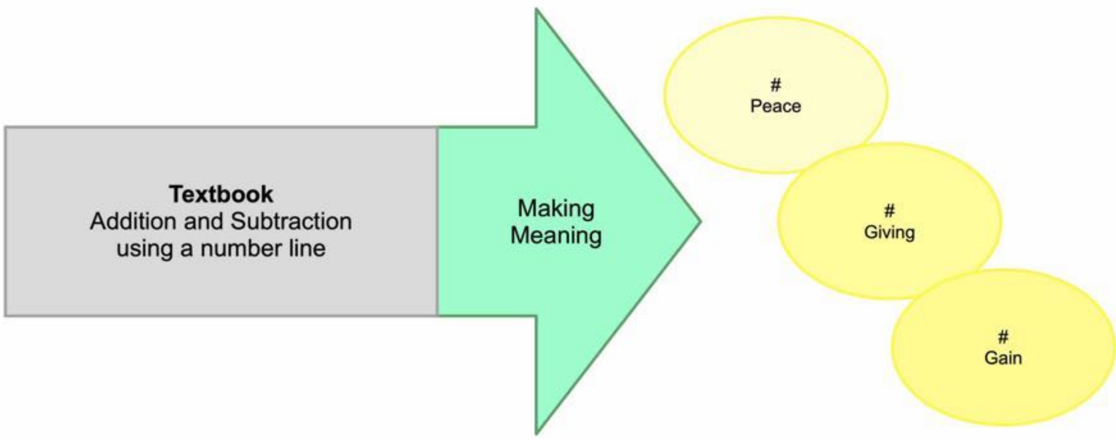

Figure 3. Representation of a values-based approach to addition and subtraction.

The end goal of the Living Mathematics: teaching pathway is to generate a life-affirming energy that teaches (Key Action 4):

Exemplar learning script using the living values peace, giving and gain

Question, what happens when numbers, like people, push in different directions?

Do they get small and sad? If 'yes' then I will never be at 'peace' (Figure 3 above) .

Answer. Sorry, well, yes! +3- 4 = -1 and -3+4 = +1. Fighting uses up a lot of energy! They could have been a 7 but they are just a 1. They are so generous in 'giving' (Figure 3) their potential value away.

Question. But when numbers, like people, work together do they get bigger and happier? If 'yes' then I will be at 'peace' forever (Figure 3).

Answer. Well, yes, you are lucky! ...

Look… +3 + 4 = +7 and – 3 – 4 = - 7

the numbers 'gain' (Figure 3) so much by pushing together to become a 7.

The moral to the story?

Work together, gain and be at peace.

Work apart and rejoice in giving to others (Williamson, 2016).

Discussion.

Peter, (Mellett, 2020) suggests that success depends on the learner 'picturing' such facts to themselves and asked 'can you do this with more than one pupil at a time?' I think the one-to-one environment gives me an opportunity to tailor learning interactions to how I perceive the learning needs of one individual, and this would include our imagining together. I think there is more risk if working in this way with a group because the creative uncertain nature of this teaching exercise may cause some stories and games to be quickly abandoned following an unsuccessful short trail. However, in a drama workshop the participants' toy values and beliefs could be shared and acted out by more than one learner at a time like an improvised surreal script of a play.

We have seen how a Dodgsonian-like pretend world could facilitate learning using a value-based approach. The next case study shows in more detail how an individual's values and beliefs can be identified in response to a mathematical form.

b) I as a teacher of morals in mathematics

I imagine being a student sitting in a mathematics lesson. The teacher hands out a worksheet to help us to practice calculating the mean of a small set of numbers. I understand what is required, find my calculator and start to enter the data hoping that I will not make any mistakes. The group of learners had not set the time aside to consider, in any detail, their reasons for wanting to map a set of numbers to only one. They had not challenged themselves to propose, and to argue for, any alternative ways of achieving this end, or not, so were they merely operating as an involuntary calculating machine seeking the teacher's validation? I then imagined taking part in a Personal Social and Emotional Development (PSED) lesson; a debate about the importance of the modern family. I did not expect there to be any correct answers and I saw this lesson as an opportunity to express my own ideas, grow my social intelligence, and learn from opposing views. This comparison of narratives suggests that juxtaposing the words ‘Living’ and ‘Mathematics’ is inappropriate and contradictory. ‘Living’ suggests personalisation; taking account of an individual’s values and beliefs as in the PSED lesson, while the mathematical task smacks of a machine-like uncreative approach.

Following my teaching pathway.

Consider the Formula (4) for calculating the mean of a set of numbers (Key Action 1). Key Action 2 of my Living Mathematics teaching pathway requires making meaning of this formula and Key Action 3 requires further a contextualisation of the student and teacher’s living values and beliefs in this content.

I can make meaning. Finding the mean value of a set of numbers using Formula (4) is sharing-out and the formula provides a simple algorithm.

Mean Value = Total Value/Number of Numbers (4)

My attempt to contextualise my living values and beliefs in this content would be to state that Formula (4) represents an intellectual straitjacket, a code that demands that anyone who reads it follows its rigorous instruction. The only way to believe that the mean of the numbers 7, 7, 9, 12 and 15 is not 10, is to be wrong. If I had refused to calculate the mean because doing so was contrary to my living value elegance, growth or integrity (Williamson, 2015, p102, 103) and my belief that mathematical thought should be liberated, then perhaps this would have facilitated more lively learning interactions in the mathematics classroom.

As a teacher, I can demonstrate how a given mathematical form has initiated my living narrative. I can describe the living contradictions I have encountered when asking questions of the kind ‘how do I as a teacher of mathematics communicate more accurately the true meaning of this formula here?‘ (Whitehead, 1989). Doing this could make the gift of a life affirming energy to an otherwise lifeless set of equations and definitions.

Calculating the mean of 7, 7, 9, 12 and 15 (as 10) is consistent with my value equality. This is because 10 is the value that all the numbers can be changed to: 10, 10, 10, 10 and 10, through the equal distribution of their size. Doing this does not alter the total (50) but makes all the numbers equal. If these individual numbers are to be represented by only one, then I believe that this number should be the mean because following the distribution of size, they all become the mean value anyway. However, doing this is contrary to my value:

1. Fairness, [Figure 4] because 15, has a greater influence over the total than the smaller values, like 7. Engaging with Formula (4) makes me experience a living contradiction with respect to my value fairness as being fair would imply that each number should be counted as one: ‘one person one vote’. Formula (4) represents a situation such as the privileged are given two votes. If the individual numbers: 7, 7, 9, 12 and 15 are to be represented by only one, then that number should be the median which is 9 because then all the numbers are counted just once, and their

size is used only to place them in order; but using the median to represent this set of numbers imposes a regime under which becoming more than just greater than, or less than just less than, has no merit.

2. Uniformity, because no recognition of the repetition of the number 7 has been acknowledged. Engaging with Formula (4) makes me experience a living contradiction with respect to my value uniformity as this expects that the quality sameness is rewarded due to its association with the admirable qualities, dependency and reliability. If these individual numbers are to be represented by only one, then that number should be the mode which is 7 because this is the only number that displays uniformity in the group.
3. Merit, because merit implies that superior magnitude should be acknowledged, as in a meritocracy. Engaging with Formula (4) makes me experience a living contradiction with respect to my value merit and I would be frustrated and offended if merit as magnitude were not given pride of place due to its association with the admirable quality rank. If these individual numbers are to be represented by only one, then that number should naturally be the highest which is 15 because this is the only number that has true merit judged by its relative size.
4. Diversity, because this implies diversity should be acknowledged and celebrated. Engaging with Formula (4) makes me experience a living contradiction with respect to my value diversity as two groups could have the same mean but one could be far more diverse, and therefore judged to be more worthy. If these individual numbers are to be represented by a subset of the group, then this should indicate to what extent the group is diverse, as this is the group's most relevant quality. The set of numbers should then be represented by a subset containing two numbers, the lowest and the highest {7, 15} because this shows the true worth of the group.
5. Inclusion, because this implies that all numbers have an equal right to represent the group. Engaging with Formula (4) makes me experience a living contradiction with respect to my value inclusion because representing a group of five numbers by just one excludes the other four. The group should not be represented by only one, and the set of numbers should only be represented by itself: {7, 7, 9, 12, 15} because this is totally inclusive.

Discussion.

It seems that this exercise has transformed a traditional mathematics lesson and lesson plan into a PSED-type exercise. I argue that such an activity has the potential to bring the student and the teacher closer to the mathematical forms being considered because if the values and beliefs someone cares about are embedded in an object to be studied then it works.

As teaching and research are closely related, complementary activities, the relevance of a researcher's values and beliefs about a mathematical form may usefully be considered. The next case study illustrates how, rather than being a catalyst for learning, an individual's values and beliefs can underpin the creation of a new mathematical form.

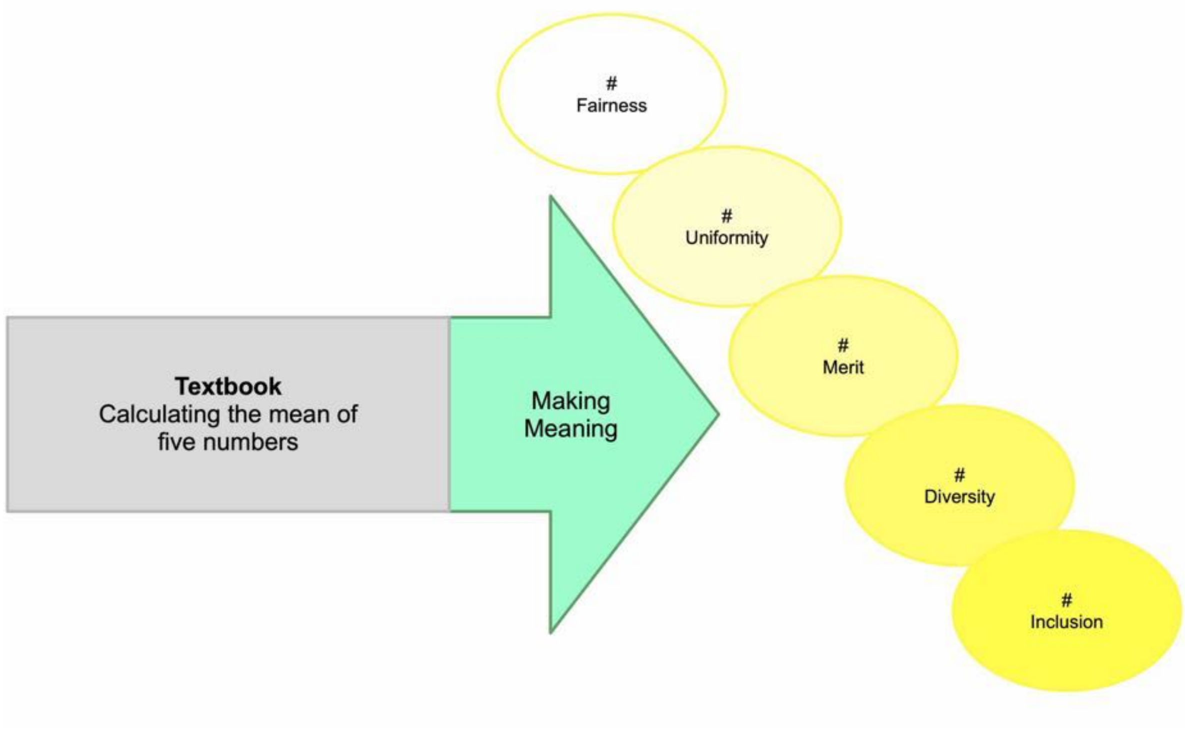

Figure 4. Representation of a value-based approach to calculating the mean of five numbers

c) I as the author of a five-cycle living visual taxonomy of learning interactions

Following my research pathway. Pip Bruce Ferguson was my Living Theory mentor in Williamson (2015) and supported my independent identification of my living values (Key Action 1). Becoming aware of the content Dewey (1897), Flanders (1970) and Markov (1971) is to perform Key Action 2. Contextualising my living values in this content (Key Action 3) was challenging at first in view of its authoritative nature. Making meaning by comparing the theoretical ideas described in the content to my lived experiences served as a starting point. The identification and application of my dominant values; comfort, elegance, growth, humanitarianism, humour, integrity, perseverance and scholarship (Figure 5) preceded the staged development of a five-cycle living visual taxonomy of learning interactions (Williamson, 2015). My values of elegance, student growth and perseverance contributed to the identification of three levels on knowhow (1) I know I know, (2) I am stuck and (3) that is remote. My reflection on learning interactions during one-to-one tuition, further reading, my proposal of a five-cycle taxonomy of learning interactions (Figure 5), an associated mathematical model and observation protocol was to perform Key Action 4.

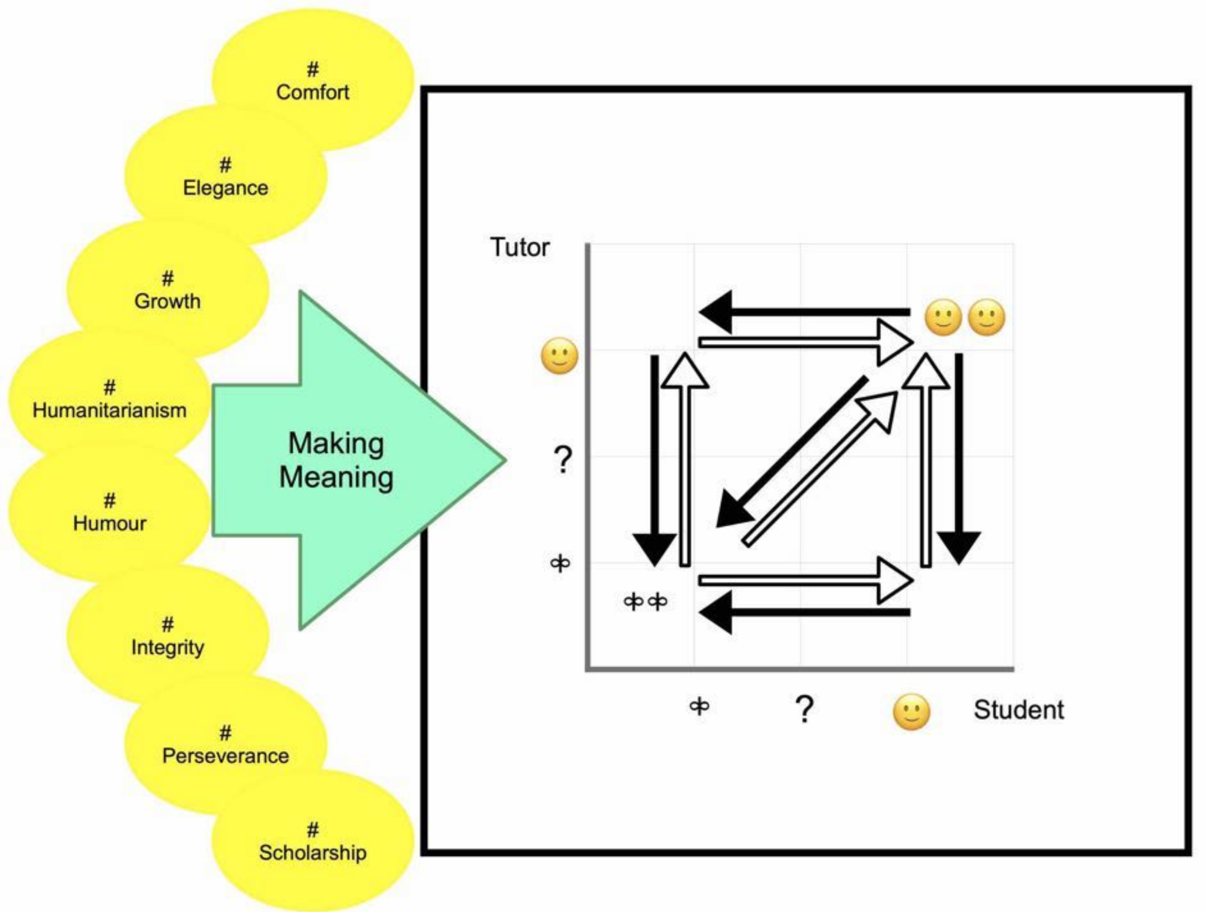

Figure 5. Representation of a value-based approach to the creation of the mathematical model: a five-cycle living visual taxonomy of learning interactions.

Discussion.

This example of Living Mathematics shows how new content can be built from living values that are thought of as axioms. This pseudo axiomatic approach mirrors a more rigorous mathematical system in which theorems are derived from a set of axioms expressed mathematically. It is the making of meaning of these 'living axioms' that lead to the creation of the new mathematical model.

The next case study illustrates how reflection on research that took place before I was aware of Living Educational Theory research may still be interpreted as a Living Mathematics research pathway.

d) I as a generalisor of formulae for estimating heritability

Following my research pathway.

In the late 1970s and early 1980s I was a research student who felt passionately about mathematics and was keen to make a contribution to knowledge. I did not know my values and beliefs by name but I sensed their presence in almost everything I did (Key Action 1). Becoming aware of the content Fujishima and Freedeen (1972) a paper titled 'general formulae for estimating heritability with related parents' was to perform Key Action 2.

Reading this paper as a novice applied science researcher with a background in pure mathematics made me feel uneasy. However, it was several decades later that I recognised this feeling as a living contradiction with respect to my value integrity. I became uneasy because Fujishima and Freedeen had omitted non-genetic terms from their equations in order to obtain their estimate of heritability (Key Action 3). This was a common and respected practice in applied mathematics but unusual in pure mathematics. I solved for heritability without omitting these terms by using a 'hidden polynomials' approach I devised (Williamson, 1984, Chapter 7). This remedied the dissonance between my values and beliefs and my actions. I had modified the published content I had read (Key Action 4) driven by a life affirming energy.

Discussion.

Living Educational Theory research was unknown to me as a research student, some time before the publication of Whitehead (1989). However, I was aware of my values elegance, integrity and perseverance.

I can rationalise my narrative here by using the taxonomy of learning interactions described in research case study (c) above (Williamson, 2015). Knocking down (p 111) and then building up (p 112) perceived levels of knowhow is a feature of my five-cycle living taxonomy of learning interactions (Figure 5) and has become a part of my living narrative. The meaning of these phrases is personal, relatable but not necessarily generalisable. The process of knocking down my own level of knowhow can be a brave step because it turns concepts and principles that I recognise as established and indisputable into tentative once more. Further, a requirement to building up my level of knowhow after it has been knocked down may be unwelcomed especially if I did not fully recognise the need for knocking it down and asking further challenging questions.

I understand that the use of a case study for mathematical models of inheritance is emotive to some educationalists who interpret this as an endorsement of questionable practices, attempts to measure the unmeasurable: intelligence, wellbeing and other aspects of human florishing. I would base this thinking on a cultural paradigm. In science, quantitative genetics is the study of many measurable traits, from body mass index to susceptibility to disease, which I believe supports our health and food security. I would base this thinking on an empiricist paradigm. I believe that the discovery of a conflict between these paradigms confirms that Living Mathematics is a generic research methodology. That is, Living Mathematics is just as applicable to 'hard science' research as it would be to the humanities. The values and beliefs held by all researchers are equally valid because they are all 'only human'.

Geneticists have debated their motive for, and ways of, generalising formula for estimating heritability. If all relevant factors were included in the models then this would be in keeping with my values integrity and elegance. The models would be comprehensive and perhaps more beautiful.

Simplifying the mathematical model in order to obtain heritability estimates made me feel uneasy because it smacked of a dogmatic machine-like thought. The only way to estimate heritability was to believe that there are no non-genetic causes of similarity. One colleague remarked 'as soon as you start making assumptions Brian the sooner you will be finding answers.' I thought, better to have no answers than wrong ones. This was my dilemma.

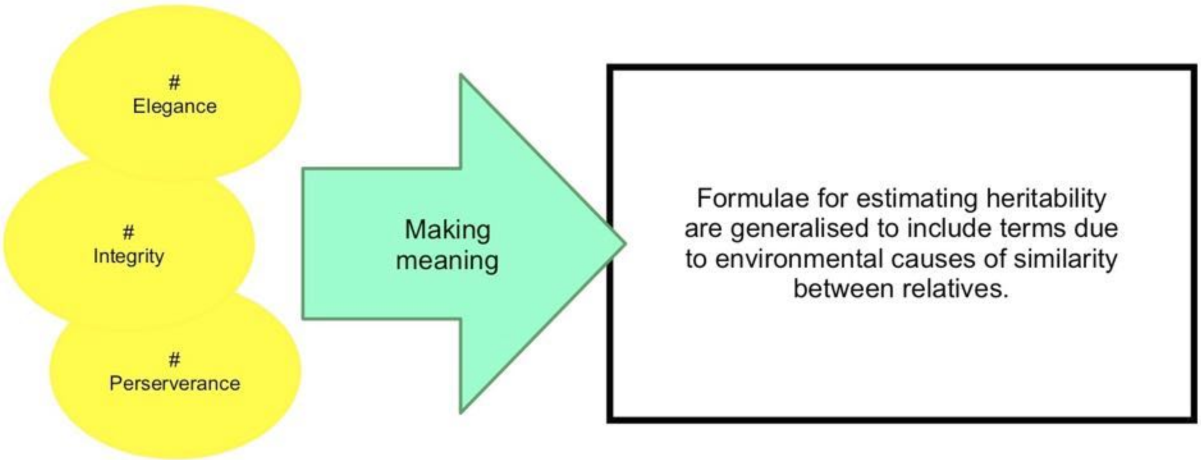

Figure 6. Representation of a value-based approach to the modification of a mathematical model

The four case studies described in this section have shown how the application of Living Educational Theory to the teaching of, and research using, mathematics can manifest itself in a range of guises.

Discussion

Mathematical thinking

The National Centre for Excellence in the Teaching of Mathematics (NCETM) identify five principles of engagement with mathematics content drawn from the literature: coherence, representation and structure, mathematical thinking, fluency and variation. This paper addresses mathematical thinking:

> "… if taught ideas are to be understood deeply they must not merely be passively received but must be worked on by the student, thought about, reasoned with and discussed with others". (NCETM, 2020)

Teaching and research pathways

The Living Mathematics teaching and research pathways defined and exemplified above offer one suggestion as to how the mathematical thinking component could be approached by students, teachers and researchers alike.

The teaching and research pathways differ from each other. In the teaching pathway the participants' values and beliefs are identified as part of the teaching and learning exercise as a reaction to the content. In the research pathway the participants' values and beliefs are identified independently of the content. Teacher or student values and beliefs may act as a teaching or learning aid. A researcher's values and beliefs may work as a catalyst for the creation of new mathematics.

The living context of a mathematical form

In the spirit of Living Mathematics and in the context of an individual's personal living mathematics, *symbolic forms may be interpreted in several alternative ways*. Oswalder writes:

> "There are different number worlds and the character of a piece of mathematics depends wholly on the culture in which it is rooted." (in Bloor, 1983, p 163)

The culture being provided by the contextualised values and beliefs held by the living 'I' . For example, are + 3 and – 4 not at peace and therefore destined to become smaller and sadder? Are unsolvable equations elegant or unsightly?

My value integrity and damned lies

My living value integrity has supported my living mathematics, however, in relation to mathematical statistics, Mark Twain is reported to have written:

> "There are three kinds of lies: lies, damned lies, and statistics!"

Lying must be in opposition to my living value of integrity. If Mark Twain is to be taken seriously then I ask, 'is statistics really like a person who lies and cheats?' Is it then no more than a virus that has entered my intellectual life, to pull the wool over my eyes as an unsuspecting untutored foot traveler? I consider statistics to be a set of mathematical forms, models and procedures that I can use to describe my object of study. In view of statistic's vulnerability, as being merely manufactured and not of nature, I decide that 'all models are wrong, but some models are useful' (Box, 2009)... but who decides which ones are useful; and how do they decide? Is it me who decides? If so, then how?

Reverence and authority

I am influenced by Sir Ronald Fisher's landmark paper founding quantitative genetics in 1918, The correlation between relatives on the supposition of Mendelian inheritance, and his many other accomplishments, for example, his designing of the mathematical formula for variance based on his observations of nature. Clearly such a formulae would not naturally evolve so must be created by people who have observed a phenomenon that they believe to be worth creating a formula for. Ronald Fisher has been hailed as probably the greatest statistician ever, however, my lived experience when encountering this work brings to the forefront my resistance to exclusivity and authority in my intellectual life. I value my independence as a thinker so must force myself away, give myself the opportunity to seek new approaches based on my living values and beliefs.

> "Change your statistical philosophy and all of a sudden different things become important, then 'laws' handed down from God are no longer handed down from God. They're actually handed down to us by ourselves, through the methodology we adopt." (Steven Goodman cited in Nuzzo (2014, p150).

What it means to be 'only human'

I have argued that those designing, and applying, these procedures may themselves have called upon their lived experiences as I did, captured them and represented them in a mathematical form. Their intention was not to set out to create such a monster perceived by many as an intransigent block of impenetrable knowledge. Further, perhaps the proposition that these mathematical architects called upon their lived experiences is supported by the bitter disagreements documented between them (Ioannidis, 2005, 2005a). Perhaps it was a living life-affirming force that drove the rift between Neyman and Pearson, and Fisher. A disagreement so strong that it influenced the day to day lives of millions of students, scholars and practitioners.

I believe that using a mathematical lens to gain an awareness of the world outside of myself, for example, the learning interactions of others or of quantitative genetics can be made more beneficial by the Living Educational Theory research methodology.

Play and drawing pictures

This article has attempted to begin an exploration of how I could improve my practice as a teacher and researcher by developing my living mathematics. I can suggest ways in which art and my living educational theories can support the ST∑@M (Science Technology Engineering Art and Mathematics) integrative education movement (Yakman, 2008) potentially empowering learners to engage with mathematics and communicating their own individualised lived experiences (Robinson, 2013) through the integration of a range of artistic medium: comic strips, graphics, short videos, monologues and duologues. In practice this involves tackling classroom organisational issues, such as student task creation and progression, managing preconceived expectations from some learners that STEM content is art-free. A brief outline of this aspect of the work can be found in, Williamson (2018).

Taking a toy to pieces

A living-theoretic reverse-engineering of mathematical concepts and methods may lead some students to a deeper understanding, and nurture within them, an empathy towards the creators of the concepts and methods they are being asked to learn. Taking a toy to pieces is a way of learning more about the toy. Concepts that are traditionally mathematically sophisticated or elegant or simple may be more clearly understood, unraveled and demystified, using the Living Educational Theory research approach.

How can I influence others to thrive in the creativity this approach to mathematical thinking may introduce to their working day as teachers and researchers? Calling upon their knowledge to identify their values and beliefs that relate to mathematical forms? For example, by imagining how the people who formalised counting, addition, subtraction, negative numbers, multiplication, division, fractions, mean, variance, linear equations, quadratic equations, changing the subject of a formula, gradient of a graph, trigonometry, differentials, sets, groups, mappings, matrices, determinants , operators, imaginary numbers and isomorphism must have felt. Their resilience in their battles to pursue their elegance could have been admirable. In practice, would it be possible to empower learners of all ages and roles using Dorothy Heathcote's 'Mantel of the Expert' approach (Heathcote, 1984) to 'show me/show me more' (Williamson, 2015).

'I' as my claim to knowledge

In this paper I have argued that a collaboration between Living Educational Theory and mathematics may enrich the applicability, validity and purposefulness of mathematical models as a creative medium and an organic tool. Further that, in so doing, it seems that such a collaboration may strengthen our claim to understanding and knowledge, that is, our epistemology.

I would like to explore further the scope and applicability of Living Mathematics. I would like to support students, teachers and researchers to work on their own living mathematics: I the student, I the teacher and I the researcher.

Finally, if the underlying living values and beliefs, the 'pseudo mathematical axioms', were altered, what then would become of the mathematical form? Would such manipulation of my living mathematics produce a new mathematical object more useful and informative than before? Perhaps, the story that numbers tell us is under our own control.

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